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Analysis of the quadrupole deformation of Δ(1232) within an effective Lagrangian model for pion photoproduction from the nucleon

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Abstract. We present an extraction of the $E2/M1$ ratio of the $\Delta(1232)$ from experimental data applying an effective Lagrangian model. We compare the result obtained with different nucleonic models and we reconcile the experimental results with the lattice QCD calculations.

PACS. 14.20.Gk Baryon resonances with $S = 0$ – 25.20.Lj Photoproduction reactions – 13.60.Le Meson production

The deformation of the nucleon and its first excitation, the $\Delta(1232)$, is a topic that has focused the attention of many researchers in the last years from both the experimental and the theoretical sides [1]. The possibility of such deformation has been studied using the $E2/M1$ ratio (EMR) of the $\gamma N \to \Delta(1232)$ transition [2]. The emission (absorption) of a photon by a spin-3/2 particle involves a magnetic dipole ($M1$) multipolarity and an electric quadrupole ($E2$) multipolarity. From experiments it is found that $E2$ is small but not zero, which evokes a deformed nucleon picture. A deviation from zero of the EMR is a clear indication of the existence of such deformation and allows to quantify it. This ratio is mainly obtained in two different ways, from nucleonic models such as quark models or lattice QCD, and from experimental data. Reconciliation of both extractions is significant in order to understand the structure of the nucleon. This work is concerned with the extraction of the intrinsic $E2/M1$ ratio of the $\Delta(1232)$ from experimental data using a reaction model.

In [3,4] we have developed a pion photoproduction model up to 1 GeV of photon energy based upon effective Lagrangians. The model follows closely the work of Garcilazo and Moya de Guerra [5] and has similarities with the work of Sato and Lee [6] as well as with other works [7–9] based on the seminal work of Peccei [10]. The reaction model allows us to isolate the contribution of the $\Delta(1232)$ —taking into account the high-energy behaviour of the tail of the resonance—, to calculate its EMR, and to compare it with the values provided by nucleonic models. In addition to Born terms (those which involve only photons, nucleons, and pions) and vector meson exchange terms (ρ and ω exchanges), the model includes all the four star resonances in the Particle Data Group (PDG) [11] up to 1.7 GeV mass and up to spin-3/2: $\Delta(1232)$, $N(1440)$, $N(1520)$, $\Delta(1620)$, $N(1650)$, and $\Delta(1700)$.

The model displays chiral symmetry, gauge invariance, and crossing symmetry, as well as a consistent treatment of the interaction with spin-3/2 particles that avoids well-known pathologies present in previous models [3,4]. The dressing of the resonances is considered by means of a phenomenological width which takes into account decays into one π, one η, and two π. The width fulfills crossing symmetry and contributes to both direct and crossed channels of the resonances.

We assume that the final-state interactions (FSI) factorize (πN rescattering) and can be included through the distortion of the πN final-state wave function. The calculation of the distortion requires one to calculate higher-order pion loops or to develop a phenomenological potential FSI model. Both approaches are far from the one we apply. The first one is overwhelmingly complex and the second would introduce additional model dependences, which are to be avoided in the present analysis, because we are mainly interested in the bare properties of the resonances. We rather include FSI in a phenomenological way by adding a phase δ_{FSI} to the electromagnetic multipoles. We determine this phase so that the total phase of the

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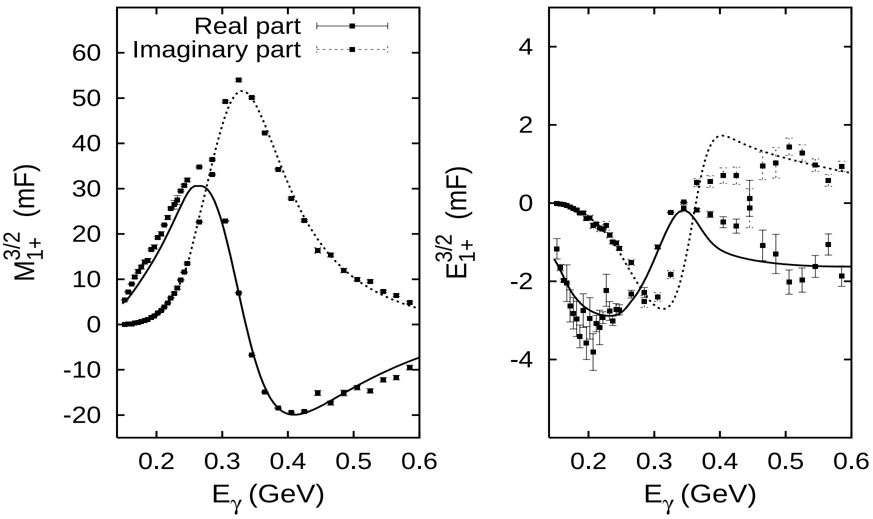

Fig. 1. $M^{3/2}_{1+}$ and $E^{3/2}_{1+}$ electromagnetic multipoles. Curve conventions: solid, real part of the multipole; dashed, imaginary part of the multipole. Data taken from [12].

electromagnetic multipole is identical to the one of the energy-dependent solution of SAID [12]. In this way we are able to isolate the electromagnetic vertex and remove the FSI effects.

In order to assess the parameters of the model we had to minimize the function χ^2 defined by

$$\chi^2 = \sum_{j=1}^{m} \left[\frac{\mathcal{M}_j^{exp} - \mathcal{M}_j^{th}(\lambda_1, \ldots, \lambda_n)}{\Delta\mathcal{M}_j^{exp}} \right]^2, \quad (1)$$

where $\mathcal{M}^{exp}$ stands for the current energy-independent extraction of the multipole analysis of SAID up to 1 GeV for E_{0+}, M_{1-}, E_{1+}, M_{1+}, E_{2-}, and M_{2-} multipoles in the three isospin channels $I = \frac{3}{2}, p, n$ for the $\gamma p \to \pi^0 p$ process [12]. $\Delta\mathcal{M}^{exp}$ is the error and $\mathcal{M}^{th}$ is the multipole given by the model which depends on the parameters $\lambda_1, \ldots, \lambda_n$, which stand for the electromagnetic coupling constants of the resonances and the cutoff Λ which regularizes the high-energy behaviour of the Born terms. The masses and the widths of the resonances have been taken from the multichannel analysis of Vrana, Dytman, and Lee [13] and from a *speed plot* calculation [4]. The EMR we present has been obtained as an average of the results obtained with both sets of masses and widths. Electromagnetic multipoles are complex quantities and we have taken into account 763 data for the real part of the multipoles and the same amount for the imaginary part. Thus, $m = 1526$ data points have been used in the fits.

In order to fit the data and determine the best parameters of the resonances we have written a genetic algorithm combined with the `E04FCF` routine from NAG libraries [14]. Although genetic algorithms are computationally more expensive than other algorithms, in a minimisation problem it is much less likely for them to get stuck at local minima than for other methods, namely gradient-based minimisation methods. Thus, in a multiparameter minimisation like the one we face here it is probably the best possibility to search for the minimum [3,15].

Table 1. Comparison of $\mathrm{EMR}^{\mathrm{physical}}$ values from experiments compared to the values obtained with reaction models.

	$\mathrm{EMR}^{\mathrm{physical}}$	Ref.
Experiments		
LEGS Collaboration	$(-3.07 \pm 0.26 \pm 0.24)\%$	[17]
A1 Collaboration	$(-2.28 \pm 0.29 \pm 0.20)\%$	[18]
A2 Collaboration	$(-2.74 \pm 0.03 \pm 0.30)\%$	[19]
Particle Data Group	$(-2.5 \pm 0.5)\%$	[11]
Reaction models		
Fernández-Ramírez *et al.*	$(-3.9 \pm 1.1)\%$	[16]
Pascalutsa and Tjon	$(-2.4 \pm 0.1)\%$	[8]
Sato and Lee	-2.7%	[6]
Fuda and Alharbi	-2.09%	[20]

In fig. 1 we show the fits to $M^{3/2}_{1+}$ and $E^{3/2}_{1+}$ electromagnetic multipoles.

Different definitions of the EMR have been employed in the literature. We should distinguish between the *intrinsic* or *bare* EMR of the $\Delta(1232)$ and the directly measured value in experiments which is often called *physical* or *dressed* EMR value [6,8,16]. The physical EMR is obtained as the ratio between the imaginary parts of $E^{3/2}_{1+}$ and $M^{3/2}_{1+}$ electromagnetic multipoles at the invariant mass (photon energy) at which $\mathrm{Re}[M^{3/2}_{1+}] = 0 = \mathrm{Re}[E^{3/2}_{1+}]$. Since all the reaction models are fitted to the experimental electromagnetic multipoles, they generally reproduce the physical EMR value within the error bars (see table 1). We obtain

$$\mathrm{EMR}^{\mathrm{physical}} = \frac{\mathrm{Im}\left[E^{3/2}_{1+}\right]}{\mathrm{Im}\left[M^{3/2}_{1+}\right]} \times 100\% = (-3.9 \pm 1.1)\%. \quad (2)$$

However, this measured EMR value is not directly available from theoretical models of the nucleon and its

Table 2. Comparison of EMR^{bare} values extracted from experiments through reaction models compared to the values obtained with nucleonic models.

	EMR^{bare}	Ref.
Reaction models		
Fernández-Ramírez *et al.*	$(-1.30 \pm 0.52)\%$	[16]
Pascalutsa and Tjon	$(3.8 \pm 1.6)\%$	[8]
Sato and Lee	-1.3%	[6]
Davidson *et al.*	-1.45%	[7]
Garcilazo and Moya de Guerra	-1.42%	[5]
Vanderhaeghen *et al.*	-1.43%	[9]
Nucleonic models		
Non-relativistic quark model	0%	[21]
Constituent quark model	-3.5%	[22]
Skyrme model	$(-3.5 \pm 1.5)\%$	[23]
Lattice QCD (Leinweber *et al.*)	$(3 \pm 8)\%$	[24]
Lattice QCD (Alexandrou *et al.*)		[25]
$(Q^2 = 0.1\,\text{GeV}^2,\ m_\pi = 0)$	$(-1.93 \pm 0.94)\%$	
$(Q^2 = 0.1\,\text{GeV}^2,\ m_\pi = 370\,\text{MeV})$	$(-1.40 \pm 0.60)\%$	

resonances. Instead, if we want to compare to models of nucleonic structure, it is necessary to extract the bare EMR value of $\Delta(1232)$ which is defined as

$$\text{EMR}^{\text{bare}} = \frac{G_E^{\Delta(1232)}}{G_M^{\Delta(1232)}} \times 100\% = (-1.30 \pm 0.52)\%, \quad (3)$$

The EMR defined in this way depends only on the intrinsic characteristics of the $\Delta(1232)$ and can thus be compared directly to predictions from nucleonic models. It is not, however, directly measurable but must be inferred (in a model-dependent way) from reaction models.

The intrinsic quadrupole deformation of the $\Delta(1232)$ is found to be EMR $= (-1.30 \pm 0.52)\%$, indicative of a small oblate deformation. In tables 1 and 2 we compare our EMR values (bare and physical) to the ones extracted by other authors using other models for pion photoproduction, as well as to predictions of nucleonic models.

Reconciliation of the experimental value of the $E2/M1$ $\gamma N \to \Delta(1232)$ transition ratio ($\text{EMR}^{\text{physical}}$) with the one obtained using lattice QCD (EMR^{bare}) within a consistent and sound framework (besides our analysis only in [8] a consistent treatment of the spin-3/2 interaction is performed) is one of the goals of this work [16]. Our results also indicate that quark models need improvements in order to reproduce the value obtained from experiment.

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